

# Traveling mirror compressor delay line with nonconstant capacitance

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The scaling relations for a traveling mirror magnetic compressor [P.M. Bellan, Phys. Rev. Lett. 43, 858 (1979)] having nonconstant capacitance are derived. Varying capacitance (rather than inductance) makes possible a lower impedance device, and hence, higher field levels or faster compression times.

Recently, the author proposed a simple new scheme<sup>1</sup> for simultaneously achieving axial translation and efficient, large compressions of reversed field configurations.<sup>2-12</sup> In this note a variation upon that scheme is presented.

In the scheme of Ref. 1 a traveling mirror was created by injecting a double humped current pulse onto a macroscopic electronic delay line, consisting of solenoid coils connected to external capacitors. By gradually changing the radius  $r$ , the number of turns  $m$ , and the intercoil spacing  $\Delta$  of successive coils in the solenoid delay line, the coil inductance was made to be a slowly increasing function of axial position  $z$ , so that the traveling mirror propagation velocity became a slowly decreasing function of  $z$ . From WKB theory this caused the propagating magnetic mirror to steepen and axially contract; i.e., the wave magnetic energy became concentrated into a smaller volume. Hence, particles confined by the traveling mirror would be compressed and heated in an extremely efficient manner.

To ensure a gradual change in impedance from coil to coil, each solenoid coil of the compressor of Ref. 1 must have a relatively large number of turns, making this compressor a relatively high impedance device. An abrupt change (e.g., what would occur in going from, say, one to two turns) would violate the WKB requirement of gradual change, i.e., that  $k^{-1}dk/dz \ll k$ , where  $k$  is the wave number of the longest Fourier component of the moving mirror.

It is shown here that by gradually increasing the capacitance (rather than the inductance) of successive delay line sections, it is possible to achieve the same compression as in Ref. 1 with a much lower impedance, and hence much lower voltage device. Because breakdown of insulation at high voltages is typically the limiting parameter for operation at high field strengths or fast compression times, lower impedance permits higher field strengths for a given voltage or, alternatively, faster compression times. A simple substitution of  $C$  into the scaling expressions of Ref. 1 will not yield the correct scaling behavior for nonconstant  $C$  because one cannot tell *a priori* which derivatives operate on  $C$ . The appropriate modification of the wave equation required to incorporate nonconstant  $C$  and the resultant new scaling laws are derived here.

Figure 1 shows the circuit of a delay line having gradual change in the magnitude of  $C$  from section to sec-

tion;  $I_n$  and  $C_n$  are the respective current in, and capacitance of the  $n$ th mesh. The magnetic field  $B$  at the  $n$ th coil is

$$B = \mu_0 m I_n / \Delta,$$

the flux linking the  $n$ th coil is

$$\Phi = \mu_0 m^2 I_n \pi r^2 / \Delta,$$

and the voltage around the  $n$ th mesh is

$$\frac{d\Phi}{dt} + \int_0^t \frac{(I_n - I_{n-1})dt}{C_n} + \int_0^t \frac{(I_n - I_{n+1})dt}{C_{n+1}} = 0. \quad (1)$$

Noting that the change in coil parameters and capacitance is small from section to section, the current and capacitance may be expanded

$$I_{n\pm 1} = I_n \pm \frac{dI_n}{dn} + \frac{1}{2} \frac{d^2 I_n}{dn^2},$$

and

$$\frac{1}{C_{n\pm 1}} = \frac{1}{C_n} - \frac{1}{C_n^2} \frac{dC}{dn}.$$

Thus, Eq. (1) becomes

$$\begin{aligned} \frac{d\Phi}{dt} + \frac{1}{C_n} \int_0^t \left( \frac{dI_n}{dn} - \frac{1}{2} \frac{d^2 I_n}{dn^2} \right) dt \\ + \left( \frac{1}{C_n} - \frac{1}{C_n^2} \frac{dC}{dn} \right) \int_0^t \left( -\frac{dI_n}{dn} - \frac{1}{2} \frac{d^2 I_n}{dn^2} \right) dt = 0, \end{aligned}$$

or, on taking a time derivative, and substituting for  $\Phi$ ,

$$\frac{\mu_0 m^2 \pi r^2}{\Delta} \frac{d^2 I_n}{dt^2} - \frac{d}{dn} \frac{1}{C} \frac{dI_n}{dn} = 0. \quad (2)$$

Equation (2) is of the form

$$a(\xi) \frac{d^2 y}{d\xi^2} - \frac{d}{d\xi} b(\xi) \frac{dy}{d\xi} = 0. \quad (3)$$

Making the usual WKB assumption that

$$y(\xi) = F(\xi) \exp \left( i \int^\xi k_n(\xi') d\xi' - i\omega t \right),$$

and substituting into Eq. (3), gives

$$k_n = \omega(a/b)^{1/2}, \quad F \sim \omega^{-1/2}(ab)^{-1/4}.$$

(Note that  $k_n$  is the wavenumber in  $n$  space rather than  $z$  space.) Using Eq. (2) and, noting that the scaling of  $\lambda_n$ , the axial wavelength of the traveling mirror in  $n$  space, corresponds to  $k_n^{-1}$ , gives

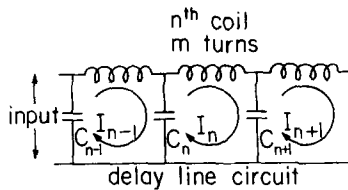


FIG. 1. Delay line circuit as in Ref. 1, except that the capacitance changes from section to section; i.e.,  $C_{n+1} \neq C_n$ .

$$\lambda_n \sim \Delta^{1/2} / m r C^{1/2}.$$

The axial wavelength  $\lambda$  in  $z$  space is obtained using the relation  $dz = \Delta dn$ , so that  $\lambda = \Delta \lambda_n$ , or

$$\lambda \sim \Delta^{3/2} / m r C^{1/2}. \quad (4)$$

Using Eq. (2), and noting that the scaling of  $I_n$  corresponds to the amplitude  $F$ , gives

$$I_n \sim (\Delta C)^{1/4} / (m r)^{1/2}.$$

Thus, the magnetic field scales as

$$B \sim m^{1/2} C^{1/4} / \Delta^{3/4} r^{1/2}. \quad (5)$$

Since the dependence on  $C$  in Eqs. (4) and (5) is always associated with  $m$  in the form  $m C^{1/2}$ , variation of  $m$  (as was described in Ref. 1) can be replaced, by variation of  $C^{1/2}$ . The validity of Eqs. (4) and (5) may be checked by showing that, as expected, the magnetic energy of the traveling field is constant

$$B^2 r^2 \lambda \sim \frac{m C^{1/2}}{\Delta^{3/2} r} r^2 \frac{\Delta^{3/2}}{m r C^{1/2}} = \text{const.}$$

In summary, Eqs. (4) and (5) show that, by making the capacitance (rather than the inductance) a slowly changing function of axial position, it is possible to achieve the same traveling, compressing magnetic field as in Ref. 1. Varying the capacitance has the advantage of making possible a much lower impedance system, because  $m$  can be reduced to being, typically, 1–3 (in-

stead of the much larger number required when the inductance is made to vary gradually). Lower impedance is of practical significance because it permits operation at higher magnetic field strengths or faster compression times for a given voltage.

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